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Energies of Friedmann–Robertson–Walker universe in $f(R, \phi, X)$ gravity

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ABSTRACT

By analyzing the energy in the sphere shell near the apparent horizon (AH), we calculate the total energy of the FRW universe and the energy of the AH in the $f(R, \phi, X)$ gravity. Then, we calculate the entropy production of the FRW universe in the $f(R, \phi, X)$ gravity. Finally, the discussion on the energy exchange between the AH and the fluid of the universe is given and the first law of thermodynamics for the AH of the FRW universe is also suggested.

1. Introduction

The thermodynamics of black holes [1–3] is proposed on the basis of the thermodynamic interpretations for the area and the surface gravity of the horizon. This indicates the deep connection between the gravity and thermodynamics. The gravitational field equations can be obtained from the entropy balance relation $\delta Q = T\,dS + T\,d_i S$ with $d_i S$ being the entropy production term [4–8]. The thermodynamics of the Friedmann–Robertson–Walker (FRW) universe in $f(R)$ gravity has been studied [9], where the first law of thermodynamics includes an entropy production term. By defining the masslike function for the FRW universe, from the Friedmann equations one derived the first law of thermodynamics in various theories of gravity [10–12]. The masslike function, including the contribution of gravitational fields, reduces to the Misner–Sharp (MS) energy at the apparent horizon (AH) of the FRW universe [10]. This treatment allows an equilibrium description of thermodynamics for the FRW universe.

The Einstein equations for spherically symmetric metric can be rewritten as the unified first law of thermodynamics [13–15]. For the FRW universe, the unified first law of thermodynamics has been studied in the Gauss–Bonnet gravity and the Lovelock gravity [16,17], the Lovelock gravity and the scalar–tensor gravity [18], the $f(R)$ gravity [19] and the Brane-World Scenario [20–22]. The Friedmann equations on the 3-brane embedded in the 5D spacetime with curvature correction terms can be written directly as the form of the first law of thermodynamics on the AH [23]. For the spherically symmetric spacetime, the MS energy is widely accepted as a well-posed quasi-local energy in Einstein gravity [24,25]. It plays a role of bridge connecting gravitational field equations to first law of thermodynamics. The generalized MS energy expression for spherically symmetric spacetime has also been obtained in Gauss–Bonnet gravity [26] and is approached by using both the integration method and the conserved charge method in $f(R)$ gravity [27].

The thermodynamical description for the FRW universe in the $f(R, \phi, X)$ gravity was studied [28–30]. The action of the $f(R, \phi, X)$ gravity includes the matter part and the coupling part between a scalar field and the curvature of spacetime. It is suggested that an equilibrium description of thermodynamics for the FRW universe in the expanding cosmological background can be appropriate [30]. This is due to the use of the same MS energy expression as that in the Einstein gravity.

In this Letter, we study the problems on the energies of the FRW universe, the energy exchange between the AH and the fluid, and the entropy production of the FRW universe in the $f(R, \phi, X)$ gravity. In Section 2, we first calculate the generalized MS energy and the total energy of the FRW universe. Then, give the formula for calculating the energy of the AH of the FRW universe. In Section 3, we first analyze the validity of the equilibrium thermodynamics for the FRW universe. Then, calculate the entropy production of the universe in the $f(R, \phi, X)$ gravity. In Section 4, we give some discussions of the energy exchange between the AH and the fluid and suggest an alternative interpretation to the energy change of the AH from the entropy production.

2. The total energy and the energy of the AH of FRW universe in $f(R, \phi, X)$ gravity

The 4-dimensional FRW universe metric reads

$$ds^2 = h_{ab}\,dx^a\,dx^b + \tilde{r}^2\,d\Omega_2^2, \tag{1}$$

where $h_{ab} = diag(-1, \frac{a^2}{1-kr^2})$ and $\tilde{r} = ar$ with $a = a(t)$ being the scale factor of the universe, $k = 1, 0, -1$ denotes the spatial curvature of the universe. From $g^{ab}\tilde{r}_{,a}\tilde{r}_{,b} = 0$, the AH radius of the FRW universe $\tilde{r}_A$ is obtained as [31]

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$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + ka^{-2}}}. \tag{2}$$

The Kodama vector for metric (1) is given by [8,32]

$$K^a = \sqrt{1 - kr^2}\big[(\partial/\partial t)^a - Hr(\partial/\partial r)^a\big]. \tag{3}$$

The 4-momentum flow measured by the Kodama observer is

$$J_a = T_{ab} K^b, \tag{4}$$

with $T_{ab} = diag(\rho_f, p_f a^2/(1 - kr^2))$ being the energy–momentum tensor of the matter. The energy flux across the sphere surface of radius $\tilde{r}$ during time interval dt is

$$\delta Q = J_a \, d\Sigma^a = \frac{4\pi \tilde{r}^2}{\sqrt{1 - kr^2}} T_{ab} K^a K^b \, dt. \tag{5}$$

From Eq. (5), the energy flux across the AH during time interval dt is obtained as [16,33]

$$\delta Q = 4\pi H \tilde{r}_A^3 (\rho_f + p_f)\, dt = -V_A \, d\rho_f, \tag{6}$$

where the continuity equation for the matter has been used.

From

$$\kappa = \frac{1}{\sqrt{-h}} \partial_a\big(\sqrt{-h} h^{ab} \partial_b \tilde{r}\big), \tag{7}$$

with h denoting the determinant of h_{ab}, the surface gravity of the AH is obtained as

$$\kappa_A = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{8}$$

Hawking temperature of the AH is proportional to the surface gravity of the AH. Another temperature of the AH is inversely proportional to the AH radius

$$T = \frac{1}{2\pi \tilde{r}_A}. \tag{9}$$

The two temperatures are different except for the de Sitter universe [16]. The former temperature appears in the united first law of the FRW universe and the latter appears in the first law of the AH of the FRW universe. In studying the generalized second law of thermodynamics for the FRW universe, the latter definition of the AH temperature is adopted [34]. Adopting the AH temperature $T = \frac{1}{2\pi \tilde{r}_A}$, Eq. (6) reduces to $\delta Q = T\, dS$ in the Einstein gravity, with $S = \pi \tilde{r}_A^2 / G$ being the entropy of the AH of universe.

The $f(R)$ gravity was first studied by Buchdahl [35]. This theory can avoid the fatal Ostrogradski instability and thus can be considered as a good gravity theory [36,37]. Consider the following action of the modified gravity including the $f(R)$ gravity [30]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R, \phi, X) + S_f(g_{\mu\nu}, \psi), \tag{10}$$

where $S_f(g_{\mu\nu}, \psi)$ is the action of the matter field ψ, $X = -(1/2)\nabla^\mu \phi \nabla_\mu \phi$ is the kinetic energy of a scalar field ϕ. For convenience, we call the modified gravity with the action (10) *the $f(R, \phi, X)$ gravity*. For the metric (1), the variation for the action (10) yields the gravitational field equations as

$$F\left(H^2 + \frac{k}{a^2}\right) = \frac{8\pi G}{3}(\hat{\rho}_d + \rho_f), \tag{11}$$

$$F\left(\dot{H} - \frac{k}{a^2}\right) = -\frac{4\pi G}{3}(\hat{\rho}_d + \hat{p}_d + \rho_f + p_f), \tag{12}$$

where ρ_f and p_f are the energy density and the pressure of the matter, $\hat{\rho}_d$ and $\hat{p}_d$ are given by

$$\hat{\rho}_d = \frac{1}{8\pi G}\left[f_{,X} X + \frac{1}{2}(FR - f) - 3H\dot{F}\right], \tag{13}$$

$$\hat{p}_d = \frac{1}{8\pi G}\left[\ddot{F} + 2H\dot{F} - \frac{1}{2}(FR - f)\right], \tag{14}$$

with $f = f(R, \phi, X)$, $F = df/dR$, $\dot{F} = \partial_t F$, $\ddot{F} = \partial_t \dot{F}$ and $f_{,X} = \partial_X f$. Here, $\hat{\rho}_d$ and $\hat{p}_d$ are only related to the energy density and the pressure of the dark component in the universe. Comparing Eqs. (11) and (12) with the Friedmann equations in the Einstein gravity with the matter and dark energy components, it can be found that the induced dark energy has the energy density and the pressure given by Eqs. (31) and (32) in Ref. [30].

The matter part associated with $S_f(g_{\mu\nu}, \psi)$ in the action (10) is a perfect fluid. The energy–momentum tensor projecting onto two-dimensional spacetime normal to the sphere surface in the FRW universe takes $T_a^b = diag(-\rho_f, p_f)$ [18]. The energy of the matter in the FRW universe can be calculated by

$$dE_{eff} = A\Psi_a \, dx^a + W \, dV, \tag{15}$$

where A and V are the area and volume of the sphere of radius $\tilde{r}$, Ψ and W are the energy supply vector and the work density. For the FRW universe, Eq. (15) may be written as

$$dE_{eff} = A(r,t)\, dt + B(r,t)\, dr, \tag{16}$$

where

$$A(r,t) = 4\pi \tilde{r}^2 \big(1 - kr^2\big)(T_{tr}\tilde{r}_{,r} - T_{rr}\tilde{r}_{,t}) = -4\pi \tilde{r}^3 H p_f, \tag{17}$$

$$B(r,t) = 4\pi \tilde{r}^2 (T_{tt}\tilde{r}_{,r} - T_{tr}\tilde{r}_{,t}) = 4\pi \tilde{r}^2 a \rho_f. \tag{18}$$

Integrating Eq. (16) gives the generalized MS energy as

$$\begin{aligned} E_{eff} &= \int B(r,t)\, dr + \int \left[A(r,t) - \frac{\partial}{\partial t}\int B(r,t)\, dr\right] dt \\ &= a^3 \int 4\pi r^2 \rho_f \, dr + \int 0 \, dt \\ &= \rho_f V + c(t), \end{aligned} \tag{19}$$

with $c(t) = 0$, which gives Eq. (5.5) of Ref. [27] in the $f(R)$ gravity. Alternatively, by calculating the energy supply vector and the work density one can also have $dE_{eff} = d(\rho_f V)$ [18]. The generalized MS energy in $f(R, \phi, X)$ gravity is nothing but the matter energy inside the sphere of radius $\tilde{r}$.

Letting $\tilde{r} = \tilde{r}_A$, Eq. (19) yields the energy of the fluid inside the AH of universe as

$$E_{fA} = \rho_f V_A = \frac{4}{3}\pi \rho_f \tilde{r}_A^3. \tag{20}$$

For the Einstein gravity, this energy is $\frac{1}{2G}\tilde{r}_A$. In order to analyze the energy transfer precess occurring near/on the AH of universe, we calculate the energy in the sphere shell of inner radii $\tilde{r}_A$ and thickness $d\tilde{r}$. At a given t, expanding E_{eff} around $\tilde{r} = \tilde{r}_A + d\tilde{r}$ gives the energy in the sphere shell as

$$dE_{eff}|_t = 4\pi \tilde{r}_A^2 \rho_f \, d\tilde{r}. \tag{21}$$

Imaging that at the subsequent time, $t + dt$, the AH of the universe increases $d\tilde{r}_A$, the energy of the fluid inside the AH increases

$$dE_{Af} = \rho_f \, dV_A + V_A \, d\rho_f. \tag{22}$$

Taking $d\tilde{r} = d\tilde{r}_A$, one can see that the first term on the right-hand side of (22) is equivalent to the energy in the sphere shell at time t. This indicates that the second term on the right-hand side of (22) should be related to the energy exchange between the fluid and the AH of the FRW universe.

A non-zero term $c(t)$ may be introduced in Eq. (19), which reflects the existence of the surface energy of the AH of the FRW universe. Define $E = E_{eff} + c(t)$, then from $\partial A(r,t)/\partial r = \partial B(r,t)/\partial t$ for $c(t) = 0$, the integrability condition for E is given as

$$\partial \tilde{A}(r,t)/\partial r = \partial \tilde{B}(r,t)/\partial t, \tag{23}$$

with $\tilde{A}(r,t) = A(r,t) + dc(t)/dt$ and $\tilde{B}(r,t) = B(r,t)$. As a result, Eq. (16) may be generalized to the following form

$$dE = \tilde{A}(r,t)\,dt + \tilde{B}(r,t)\,dr. \tag{24}$$

Let $c(t) = E_{AH}$, then E is expressed as

$$E = E_{eff} + E_{AH}, \tag{25}$$

where E denotes the total matter energy inside the sphere surface of radius $\tilde{r} > \tilde{r}_A$ and E_{AH} is the surface energy of the AH of the universe.

The surface energy of the AH is a function of time and thus can be written as $E_{AH} = E_{AH}(\tilde{r}_A)$. Letting $\tilde{r} = \tilde{r}_A$ and defining $E_T = E(\tilde{r}_A)$, Eq. (25) gives

$$E_T = E_{fA} + E_{AH}, \tag{26}$$

which is the total matter energy of the universe with the AH as the boundary. Differentiating E_T yields

$$dE_T = \rho_f\,dV_A + V_A\,d\rho_f + dE_{AH}. \tag{27}$$

In the process that the AH radius increases $d\tilde{r}_A = d\tilde{r}$, one can assume that $dE|_t = dE_T$. Consider Eq. (21), then Eq. (27) gives

$$dE_{AH} = -V_A\,d\rho_f. \tag{28}$$

In the process that $\tilde{r}_A$ increases with time, ρ_f decreases and thus E_{AH} increases. Integrating (28) gives the formula for calculating the energy of the AH as

$$E_{AH} = -\int V_A\,d\rho_f. \tag{29}$$

In the Einstein gravity, putting $\rho_f = \frac{3}{8\pi G\tilde{r}_A^2}$ in Eq. (29) gives $E_{AH} = \tilde{r}_A/G$, which is twice large as the energy of the fluid inside the AH.

3. The entropy production in $f(R,\phi,X)$ gravity

The matter (the energy density ρ_f and the pressure p_f) associated with $S_f(g_{\mu\nu},\psi)$ in the action (10) satisfies the continuity equation

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0. \tag{30}$$

For the effective dark component (the energy density $\hat{\rho}_d$ and the pressure $\hat{p}_d$) there is [30]

$$\dot{\hat{\rho}}_d + 3H(\hat{\rho}_d + \hat{p}_d) = \frac{3}{8\pi G}\left(H^2 + \frac{k}{a^2}\right)\dot{F}. \tag{31}$$

Defining effectively the total energy density and the pressure of the universe as

$$\rho_t = F^{-1}(\hat{\rho}_d + \rho_f), \qquad p_t = F^{-1}(\hat{p}_d + p_f), \tag{32}$$

which are equivalent to those given in Eqs. (31) and (32) of Ref. [30], Eqs. (11) and (12) reduce to the standard form of the Friedmann equations and thus there is

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0. \tag{33}$$

For the total energy density ρ_t and pressure p_t, the equilibrium thermodynamics is appropriate. From Eq. (32), it follows that the energy density and the pressure of the induced dark energy may also be written as $\rho_d = \rho_t - \rho_f = F^{-1}\hat{\rho}_d + (F^{-1} - 1)\rho_f$ and $p_d = p_t - p_f = F^{-1}\hat{p}_d + (F^{-1} - 1)p_f$, which give $\rho_d = \hat{\rho}_d$ and $p_d = \hat{p}_d$ for $F = 1$ (the Einstein gravity). This indicates that the $f(R,\phi,X)$ gravity can give a theoretically wider description for the dark energy equation of state.

In the Einstein gravity, the Bekenstein–Hawking entropy of the AH of the FRW universe is $S = A/(4G)$ with A being the AH area [1]. In the $f(R,\phi,X)$ gravity, one should adopt the Wald entropy [38,39]

$$S = \frac{FA}{4G}, \tag{34}$$

which is consistent with the Bekenstein–Hawking entropy in Einstein gravity. In the modified gravity, the Clausius relation is modified to $dS = \delta Q/T + d_iS$ [5], with d_iS being an entropy production due to the bulk viscosity. The entropy production is thought of as a universal property of non-equilibrium thermodynamics [6]. In this sense, the modified gravity can be connected with the non-equilibrium thermodynamics.

In the $f(R)$ gravity and the scalar–tensor gravity, the equilibrium description of thermodynamics for the FRW universe can be appropriate by adopting the Bekenstein–Hawking entropy and the masslike function [10]. Whether in the $f(R,\phi,X)$ gravity the equilibrium description of thermodynamics for the FRW universe is adopted or not depends on the definitions of the AH entropy and the energy inside the AH. Using the Bekenstein–Hawking entropy $S = \frac{A}{4G}$ and the Misner–Sharp energy $E = \frac{1}{2G}\tilde{r}_A = f(\rho_d + \rho_f)V_A$ with $V_A = \frac{4}{3}\pi\tilde{r}_A^3$ leads to an equilibrium description. Adopting the Wald entropy $S = \frac{F}{4G}A$ and the MS energy $E_F = \frac{1}{2G}F\tilde{r}_A = (\hat{\rho}_d + \rho_f)V_A$, the entropy production term appears in the thermodynamic identity shown in Eq. (27) of Ref. [30].

In the $f(R)$ gravity, the generalized MS energy formula is obtained by using the integration method (the integrable condition) and the conserved charge method [27]. The generalized MS energy may be rewritten as $E_{eff} = \tilde{\rho}V_A$, where $\tilde{\rho}$ is the energy density of the matter [27]. As shown in Eq. (20), the generalized MS energy in the $f(R,\phi,X)$ gravity is written as $E_{eff} = \rho_f V_A$. It can be seen that there are the three different energy expressions E, E_F and E_{eff} for the FRW universe. For the purpose of discussing the problem on the entropy production in the $f(R,\phi,X)$ gravity, the third energy expression will be used. In what follows, we calculate the entropy production of the FRW universe in the $f(R,\phi,X)$ gravity by using the Wald entropy and the generalized MS energy. From Eq. (11) one has

$$d\rho_f = \frac{3}{8\pi G}\tilde{r}_A^{-2}\left(dF - 2F\tilde{r}_A^{-1}\,d\tilde{r}_A\right) - d\hat{\rho}_d, \tag{35}$$

where

$$d\hat{\rho}_d = \frac{1}{8\pi G}\left[(f_{,X}X)^{\cdot} + \frac{1}{2}\dot{F}R - 3\dot{H}\dot{F} - 3H\ddot{F}\right]dt, \tag{36}$$

with $(f_{,X}X)^{\cdot} = \partial(f_{,X}X)/\partial t$. Substituting it into Eq. (28) yields

$$dE_{AH} = T\,dS + d\tilde{E}_{AH}, \tag{37}$$

with

$$d\tilde{E}_{AH} = -\frac{1}{2G}\tilde{r}_A^3\left[H\ddot{F} + \frac{k}{a^2}\dot{F} - \frac{1}{3}(f_{,X}X)^{\cdot}\right]dt. \tag{38}$$

Providing that $d\tilde{E}_{AH}$ is the energy contribution coming from the entropy production, $d_iS = d\tilde{E}_{AH}/T$ yields the entropy production in $f(R,\phi,X)$ gravity as

$$d_i S = -\frac{\pi}{G}\tilde{r}_A^4\left[H\ddot{F} + \frac{k}{a^2}\dot{F} - \frac{1}{3}(f_{,X}X)^{\cdot}\right]dt, \tag{39}$$

which gives the entropy production in $f(R)$ gravity [27] for $\phi = const$. In the $f(R)$ gravity, the entropy production depends on both $\ddot{F}$ and $\dot{F}$. For Einstein gravity ($f(R) = R$), the entropy production automatically vanishes. For the spatially flat FRW universe, the entropy production depends only on $\ddot{F}$. This means that the entropy production may be zero in a special case of the $f(R)$ gravity.

4. Discussions

Comparing Eq. (6) with (28) gives $\delta Q = dE_{AH}$. It is the energy flowing into the AH during dt and thus doesn't actually pass through the AH. For the system with the AH as the boundary of the FRW universe, this reflects the process of the energy exchange between the boundary and the fluid inside it.

The total energy of the system with the AH boundary is given by (25). When the AH radius increases $d\tilde{r}_A$ during dt, the total energy increases $dE_T = \rho_f\, dV_A$. The first and second terms in the right-side hand of (22) denote the energy increases due to volume increase dV_A and density increase $d\rho_f$, respectively. In order to understand how energy transfer occurs in this process, let us divide it into the two sub-processes. In one sub-process, energy density holds unchanged and volume increases dV_A. In the other sub-process volume holds unchanged and energy density increases $d\rho_f$. The second term reflects the energy exchange between the boundary and the fluid, that is, the energy $V_A\, d\rho_f$ ($-V_A\, d\rho_f$) flows from the AH (the fluid) into the fluid (the AH) during dt.

The entropy production shown in (39) can return to that in $f(R)$ gravity [27], but it is different from (28) in Ref. [30]. This attributes to the different definitions of the energy inside the AH. The spherically symmetric horizon in a spacetime has the same temperature and the uniform matter distribution, which implies that the AH of the FRW universe (a two-dimensional system) can be in a thermal equilibrium state. This leaves the room for a reinterpretation for the energy $d\tilde{E}_{AH}$ given by Eq. (38).

It is suggested that the energy $d\tilde{E}_{AH}$ should be related to the surface tension of the AH coming from a non-vanishing shear on it [6]. The non-vanishing AH surface tension reflects that the particles inhibiting on it interact each other. When the AH changes, there will be the work done by the surface tension. Writing $d\tilde{E}_{AH} = \gamma\, dA$ with γ denoting surface tension, we can interpret it as a work term. In this sense, Eq. (37) plays a role of the first law of thermodynamics for the AH of the FRW universe in the $f(R, \phi, X)$ gravity, $dE_{AH} = dQ_{AH} + dW_{AH}$, where $dQ_{AH} = T\, dS$ is the heat associated with the AH entropy change and $dW_{AH} = d\tilde{E}_{AH}$ denotes the work done by the surface tension of the AH of the FRW universe. In the Einstein gravity, this work term vanishes.

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